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A Calculating Device for Teaching Elementary Arithmetic

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My teaching experience has been in primary and secondary schools in expanding areas of the New Towns of Harlow and Bracknell. A steady influx of children, some of whom are weak in arithmetic, presents a problem. In the Autumn of 1961 I started to design a very simple calculating machine around which I intended to build a course in elementary arithmetic. The resulting calculator, and its main applications, is described below.

Referring to the first illustration it can be seen that there is a plastic box (1) containing a dial (2) which has twenty dial holes (3). An arc of fourteen of these dial holes is visible through a gap (4) in the lid of the box. The central twelve of these visible dial holes resemble coloured counters, for the base beneath them is a contrasting colour to the rest of the calculator. These counters are clearly numbered, and a plus sign (5) and an arrow (6) help to indicate that to add, say, 8 counters, the operator should place a finger on the eighth counter and dial in the clockwise direction. Totals of numbers dialled in this way appear in the answer window (7). The tens figures which appear serially in the answer window range from 1 to 24 with an additional blank or zero position. These tens figures may be set independently with the broad tens knob (8) which moves into each setting with a firm clicking action. When the mark (9) on the tens knob is opposite the answer window the blank tens position is in the answer window. The mechanism of the calculator is not damaged if the child holds the tens knob firmly and then dials to see what happens.

When the child wishes to subtract numbers from the total shown in the answer window, he pushes the component (10) in the anti-clockwise direction so that it is hidden by the lid. The movement brings about several changes in the appearance of the calculator. Firstly the black set of figures by the "counters" are replaced by a red set which number the counters in the opposite direction. A red minus sign appears in place of the plus sign and the direction of the arrow is reversed. Finally a part of the component which selectively displays these two sets of data appears at the point (11). (Pushing this part of the component in the clockwise direction would cause the calculator to be ready for addition again.) To subtract, the operator dials in the anti-clockwise direction which is indicated by the red arrow. Other features of the design, including the use of the tally (12) down the side of the apparatus, are described later. The addition sum 8 + 5 will be described in detail to illustrate the use of the calculator. When the calculator has been adjusted so that the answer window indicates zero, eight counters are "pulled towards" the operator. The phrase "pulled towards" is easily related to the idea of addition. The dialling action is smooth and silent with the result that there is no tendency to count in units. The visual display presented by the calculator contains only such data as is necessary for simple addition, and no fine degree of physical co-ordination is required to operate the calculator. As a result the child can direct his attention to what is happening in the answer window when he is dialling. A further five counters are then added. When the total reaches ten, a click is heard. A click whenever a tens figure changes helps stress place value. Finally, having taken 8 counters and 5 counters, the child sees the total 13, in the answer window. The sum could have been worked with actual counters before working it out on the calcu-

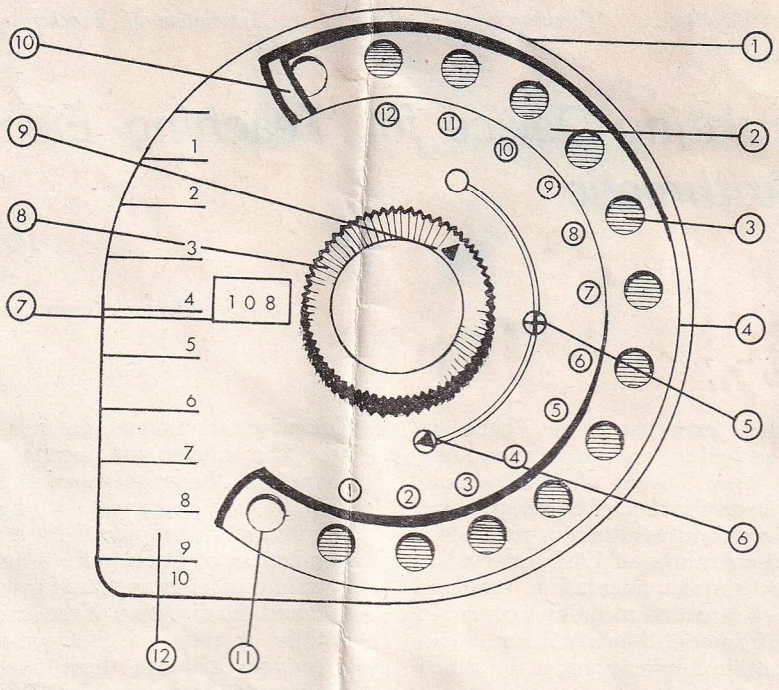

Fig. 1. The Calculator as it appears when used for addition

lator. The distances moved by the dialling finger in the above example were in the ratio of eight to five. The dial is nearly five inches across so that the differences travelled by the dialling finger when dialling different numbers is appreciable, thereby giving a clear impression of the relative number magnitudes. If the operator dials in a direction which is contrary to that indicated by the arrow, his finger will eventually come into contact with the component visible at the point (10). This will cause the minus setting of the calculator to appear. The feel and sight of the change serves to give warning of an error.

If the child dials with his finger crooked under the outer edge of a dial hole he will feel a number of flicks equal to the number dialled. These flicks are produced by small projections from the base. Normally, dialling is done smoothly and comfortably in one continuous movement—but these projections can be used when discussing the idea of a number being composed of a number of units.

Besides the relating of operations performed on the calculator to practical work, attention should, I believe, be directed to the actual words that the child uses to himself as he operates the calculator. The child should, for instance, quickly form the habit of trying to "beat the calculator". This means that each answer should be formulated before dialling. The following exercise indicates the way in which such a habit may be formed.

"Set the calculator so that the answer window shows the total 7. Now dial 4 over and over again. Whisper each answer to your partner before you dial".

By changing the two numbers in the above example, it is possible to revise all addition bonds systematically. The child should simply whisper each answer and should try not to use words such as "and four makes" between items. Such words are not needed once the brain is set to add.

If a child frequently forgets to carry appropriate figures in addition sums, the following may be tried. Take the example

46 + 27. The number 46 is set in the answer window and then 7 units added. There will be a click as a ten is "carried" to the next column. Finally the two tens of the 27 are clicked on. The sum may then be worked on the board in the usual way with a click of the fingers and a remark by the teacher when the ten is carried. The subtraction bonds up to 10 and later up to 20 may be studied. To subtract 8 from 15 the latter number is set in the answer window and then, having changed to the minus setting, 8 counters are "pushed away" from the operator. Later examples such as 246 −7 −7 −7 . . . can be selected to practice all subtraction bonds.

Multiplication is performed by continuous addition. The left forefinger keeps tally of the number of repeated diallings using the series of numbered notches down the side of the calculator. For example to find 3 × 8 the child would:—

Place the left forefinger in notch one and dial 8 with the right hand.

Place the left forefinger in notch two and dial 8 with the right hand.

Place the left forefinger in notch three and dial 8 with the right hand.

The answer, 24 appears in the answer window. When working through multiplication tables in this way the child will find that he is automatically saying his tables to himself. This method of getting children to say their tables has a number of advantages over the chanting of tables. The child works at his own pace, any error is brought to his notice, he is engaged in a meaningful sequence of actions and the design features of the calculator give complementary number information through the senses of sight, touch, hearing and movement.

Operations performed on the calculator can be related to number patterns, (a good reference for such patterns, is the first chapter of "Mathematics in the Making" by Lancelot Hogben). The second illustration shows part of the pattern of one hundred coloured counters arranged in a grid. Having found 3 × 8 = 24 with the calculator, the child may place an ⌋ shaped piece of card in the position indicated by the dotted lines to give a display of 24 counters, in three rows of eight. (*Fig.* 2)

The use of two calculators, placed side by side, is useful when discussing pairs of factors such as 3 × 8 and 6 × 4.

Division is tackled by continued subtraction. For instance, to divide 26 by 6 the number 26 is set in the answer window and the calculator prepared for subtraction. The left forefinger works the tally, and the right hand subtracts successive sixes.

The left forefinger moves to notch one and 6 is subtracted.

The left forefinger moves to notch two and 6 is subtracted.

The left forefinger moves to notch three and 6 is subtracted.

The left forefinger moves to notch four and 6 is subtracted.

At this stage the left forefinger is in notch 4 and the answer window shows that there are two left over so 26 ÷ 6 = 4 remainder 2.

Besides the prime function of giving practice with the basic number relationships, the calculator also helps with a variety of arithmetical topics.

Conversion problems can be related to the calculator. An obvious example is the conversion, up or down, of shillings and pence. The word *shilling* is written in pencil by the tally, and the word *pence* above the answer window; the child equates the counters with pence. To change 3/6 to pence the child would dial 12 three times using the tally, and then dial a 6. To change 42d. to shillings and pence the number 42 would be set in the answer window, and a division by 12 will result in the left forefinger indicating 3 shillings, and the answer window showing the remaining 6d.

Other conversion sums can be related to the calculator. For instance, the containers which children use to become familiar with gallons, quarts, and pints, may be marked with 8, 2 and 1 counters respectively. This will allow the underlying arithmetic to be related to the calculator before bookwork without its help.

When finding the area of simple rectangles e.g. a rectangle 3 in. by 8 in. the rectangle may be divided into square inches and each square inch marked with a counter. There will be an immediate resemblance to the pattern the child already uses for illustrating 3 × 8.

When the children are really familiar with the use of the calculator for the applications suggested above, the calculator can be

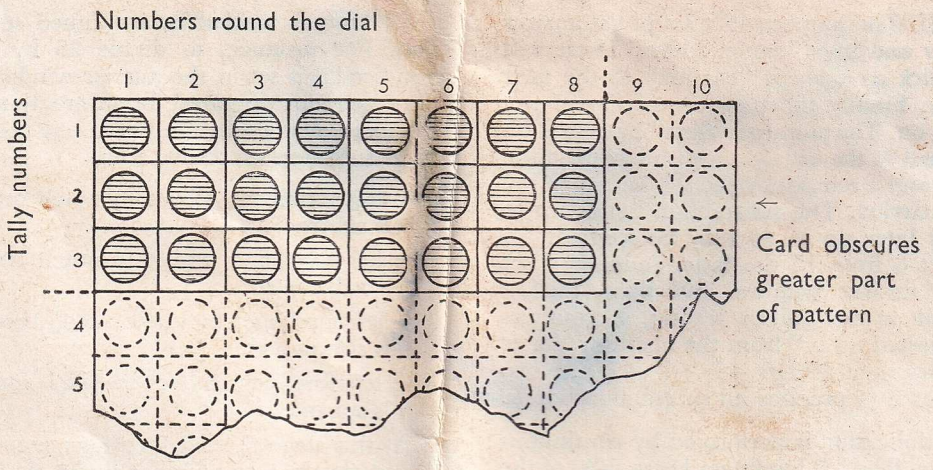

Fig. 2. Showing part of the pattern of 100 coloured "counters" used in conjunction with the calculator. The pattern visible illustrates the product of 3 and 8.

modified to help with the teaching of decimals. Each child should alter his own calculator in the following ways. Firstly, a nought is placed in the blank tens position which appears in the answer window. This is done with a soft black pencil. Secondly, the words "units", "tenths", and "hundredths" are placed above the answer window in positions corresponding to the former hundreds, tens, and units positions respectively. Thirdly, the word "tenths" is written on the central knob, and the word "hundredths" by the dial holes. Finally, a decimal point is placed between the units and tenths positions in the answer window. This is done by marking the decimal point on a piece of sellotape and sticking this across the answer window.

A square of card having the same dimensions as the square which contains the pattern of one hundred counters, mentioned previously, is cut out. This square represents unity. Each counter in the pattern occupies one hundredth of the square; each row, or column, of ten counters occupies one tenth of the square.

The calculator can now be set to zero and one hundredth dialled. This will be registered as ·01. Adding a further nine hundredths gives ·10. or one tenth. Clicking on nine further tenths give 1·00. Once place value has been taught with such examples, the calculator can be used to give a useful range of illustrative examples to introduce the rules used when working out sums involving decimals. Examples such as $1{\cdot}97 + {\cdot}3$ and $2 - {\cdot}07$ can introduce the formal rules for the addition and subtraction of decimals. The fact that the action of adding a tenth and a hundredth are different, helps to keep errors involving place value to a minimum.

Multiplication of decimals by whole numbers follows. Using the tally in the way described previously, $3 \times {\cdot}08$ is found to be ·24; $9 \times {\cdot}12$ to be 1·08. Division of decimals can be introduced by examples such as $\cdot 27 \div \cdot 09$ (for it is found that ·09 can be subtracted three times from ·27). Further examples can include $\cdot 3 \div \cdot 06$ and $1{\cdot}08 \div {\cdot}12$.

Two calculators, one of which is modified for decimals, may be used side by side when discussing percentages.

Prototypes of the calculator, upon which this article is based, were made from plastic tea-trays and perspex. My father, Mr. A. J. L. Clark who is head of the Pure Mathematics department of Plaistow Grammar School, helped in the making of these. The calculator is, however, being made available by Addo Limited under the title "The Addo Primary Calculator".

This article has attempted to show that a calculator for the early stages of arithmetic can have the function of relating simple practical activity and number patterns to number symbols in a direct and simple way.